2-1 INTRODUCTION

In this chapter we shall examine processes in which radiation interacts with matter. Three processes (the photoelectric effect, the Compton effect, and pair production) involve the scattering or absorption of radiation in matter. Two processes (bremsstrahlung and pair annihilation) involve the production of radiation. In each case we shall obtain experimental evidence that radiation is particlelike in its interaction with matter, as distinguished from the wavelike nature of radiation when it propagates. In the following chapter we shall study a generalization of this result, due to de Broglie, which leads directly into quantum mechanics. Some of the material of these two chapters may be a review of topics the student has already come across in studying elementary physics.

2-2 THE PHOTOELECTRIC EFFECT

It was in 1886 and 1887 that Heinrich Hertz performed the experiments that first confirmed the existence of electromagnetic waves and Maxwell's electromagnetic theory of light propagation. It is one of those fascinating and paradoxical facts in the history of science that in the course of his experiments Hertz noted the effect that Einstein later used to contradict other aspects of the classical electromagnetic theory. Hertz discovered that an electric discharge between two electrodes occurs more readily when ultraviolet light falls on one of the electrodes. Lenard, following up some experiments of Hallwachs, showed soon after that the ultraviolet light facilitates the discharge by causing electrons to be emitted from the cathode surface. The ejection of electrons from a surface by the action of light is called the *photoelectric effect*. It is the phenomenon underlying the operation of the solar cells being developed to convert thermal energy received from the sun directly into electrical energy.

Figure 2-1 shows an apparatus used to study the photoelectric effect. A glass envelope encloses the apparatus in an evacuated space. Monochromatic light, incident through a quartz window, falls on the metal plate A and liberates electrons,

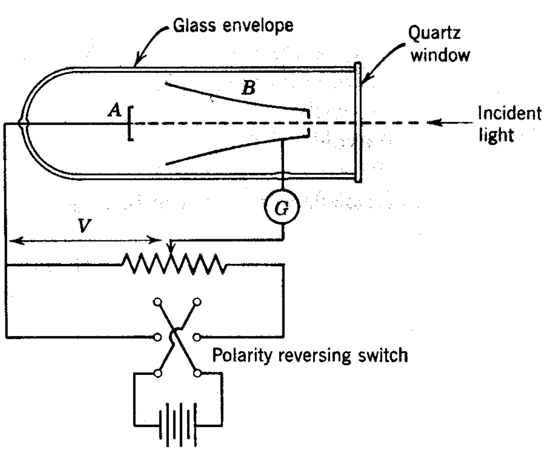

Figure 2-1 An apparatus used to study the photoelectric effect. The potential difference V can be varied continuously in magnitude, and also reversed in sign by the switching arrangement. If the same metal is used to make plate A and cup B then the potential difference between them equals the value of V measured with a voltmeter between the points indicated in the figure. But if this is not the case then the measured value of V must be corrected by adding to it the *contact potential* acting between the two metals in order to obtain the quantity of interest—the potential difference between A and B. The phenomenon of contact potential is explained in Chapter 11.

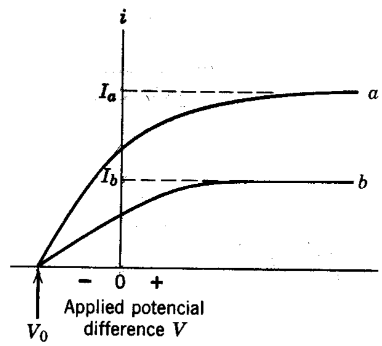

Figure 2-2 Graphs of current i as a function of potential difference V from data taken with the apparatus of Figure 2-1. The applied potential difference V is called positive when the cup B in Figure 2-1 is positive with respect to the photoelectric surface A. In curve b the incident light intensity has been reduced to one-half that of curve a. The stopping potential V_0 is independent of light intensity, but the saturation currents I_a and I_b are directly proportional to it.

called *photoelectrons*. The electrons can be detected as a current if they are attracted to the metal cup B by means of a potential difference V applied between A and B. The sensitive ammeter G serves to measure this photoelectric current.

Curve a of Figure 2-2 is a plot of the photoelectric current, in an apparatus like that of Figure 2-1, as a function of the potential difference V. If V is made large enough, the photoelectric current reaches a certain limiting (saturation) value at which all photoelectrons ejected from A are collected by cup B.

If V is reversed in sign, the photoelectric current does not immediately drop to zero, which suggests that the electrons are emitted from A with kinetic energy. Some will reach cup B in spite of the fact that the electric field opposes their motion. However, if this reversed potential difference is made large enough, a value V_0 called the *stopping potential* is reached at which the photoelectric current does drop to zero. This potential difference V_0, multiplied by electron charge, measures the kinetic energy K_{max} of the *fastest* ejected photoelectron. That is

$$K_{max} = eV_0 \tag{2-1}$$

The quantity K_{max} turns out experimentally to be independent of the intensity of the light, as is shown by curve b in Figure 2-2 in which the light intensity has been reduced to one-half the value used in obtaining curve a.

Figure 2-3 shows the stopping potential V_0 as a function of the frequency of the light incident on sodium. Note that there is a definite *cutoff* frequency ν_0, below which no photoelectric effect occurs. These data were taken in 1914 by Millikan whose painstaking work on the photoelectric effect won him the Nobel prize in 1923. Because the photoelectric effect for visible or near-visible light is largely a surface phenomenon, it is necessary in the experiments to avoid oxide films, grease, or other surface contaminants.

There are three major features of the photoelectric effect that cannot be explained in terms of the classical wave theory of light:

1. Wave theory requires that the oscillating electric vector $\mathbf{E}$ of the light wave increase in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is $e\mathbf{E}$, this suggests that the *kinetic energy* of the photo-

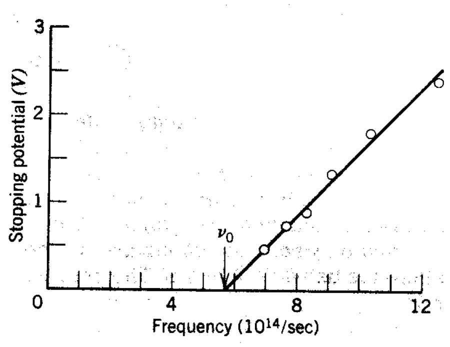

Figure 2-3 The stopping potential at various frequencies for sodium. The points show Millikan's data, except that the correction mentioned in the caption to Figure 2-1 has been recalculated using a recent measurement of the contact potential. The cutoff frequency ν_0 is 5.6×10^{14} Hz.

electrons should also increase as the light beam is made more intense. However, Figure 2-2 shows that K_{max}, which equals eV_0, *is independent of the light intensity.* This has been tested over a range of intensities of 10^7.

2. According to the wave theory the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to give the energy needed to eject the photoelectrons. However, Figure 2-3 shows that there exists, for each surface, *a characteristic cutoff frequency* ν_0. *For frequencies less than* ν_0, *the photoelectric effect does not occur, no matter how intense the illumination.*

3. If the energy acquired by a photoelectron is absorbed from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited, and probably not much more than that of a circle having about an atomic diameter. In the classical theory the light energy is uniformly distributed over the wave front. Thus, if the light is feeble enough, there should be a measurable time lag, which we shall estimate in Example 2-1, between the time when light starts to impinge on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it has accumulated enough to escape. *However, no detectable time lag has ever been measured.* This disagreement is particularly striking when the photoelectric substance is a gas; under these circumstances collective absorption mechanisms can be ruled out and the energy of the emitted photoelectron must certainly be soaked out of the light beam by a single atom or molecule.

Example 2-1. A potassium plate is placed 1 m from a feeble light source whose power is 1 W = 1 joule/sec. Assume that an ejected photoelectron may collect its energy from a circular area of the plate whose radius r is, say, one atomic radius: $r \simeq 1 \times 10^{-10}$ m. The energy required to remove an electron through the potassium surface is about 2.1 eV = 3.4×10^{-19} joule. (One electron volt = 1 eV = 1.60×10^{-19} joule is the energy gained by an electron, of charge 1.60×10^{-19} coul, in falling through a potential drop of 1 V.) How long would it take for such a target to absorb this much energy from the light source? Assume the light energy to be spread uniformly over the wave front.

▶The target area is $\pi r^2 = \pi \times 10^{-20}$ m^2. The area of a 1 m sphere centered on the source is $4\pi(1 \text{ m})^2 = 4\pi$ m^2. Thus if the source radiates uniformly in all directions (i.e., if the energy is uniformly distributed over spherical wave fronts spreading out from the source, in agreement with classical theory) the rate R at which energy falls on the target is given by

$$R = 1 \text{ joule/sec} \times \frac{\pi \times 10^{-20} \text{ m}^2}{4\pi \text{ m}^2} = 2.5 \times 10^{-21} \text{ joule/sec}$$

Assuming that all this power is absorbed, we may calculate the time required for the electron to acquire enough energy to escape; we find

$$t = \frac{3.4 \times 10^{-19} \text{ joule}}{2.5 \times 10^{-21} \text{ joule/sec}} = 1.4 \times 10^2 \text{ sec} \simeq 2 \text{ min}$$

Of course, we could modify the preceding picture to reduce the calculated time by assuming a larger effective target area. The most favorable assumption, that energy is transferred by a resonance process from light wave to electron, leads to a target area of λ^2, where λ is the wavelength of the light, but we would still obtain a finite time lag which is well within our ability to measure experimentally. (For ultraviolet light of $\lambda = 100$ Å, for example, $t \simeq 10^{-2}$ sec.) However, no time lag has been detected under any circumstances, the early experiments setting an upper limit of 10^{-9} sec on any such possible delay! ◀

2-3 EINSTEIN'S QUANTUM THEORY OF THE PHOTOELECTRIC EFFECT

In 1905 Einstein called into question the classical theory of light, proposed a new theory, and cited the photoelectric effect as one application that could test which theory was correct. This was many years before Millikan's work, but Einstein was influenced by Lenard's experiment. As we have mentioned, Planck originally restricted his concept of energy quantization to the radiating electron in the walls of a blackbody cavity. Planck believed that electromagnetic energy, once radiated, spreads through space like water waves spread through water. Einstein proposed instead that radiant energy is quantized into concentrated bundles which later came to be called *photons.*

Einstein argued that the well-known optical experiments on interference and diffraction of electromagnetic radiation had been performed only in situations involving *very* large numbers of photons. These experiments yield results which are averages of the behaviors of the individual photons. The presence of the photons is not apparent in them any more than the presence of individual droplets of water is apparent in a fine spray from a garden hose, if the number of droplets is very high. Of course the interference and diffraction experiments definitely show that photons do not travel from where they are emitted to where they are absorbed in the simple ways that classical particles, like water droplets, do. They travel like classical waves, in the sense that calculations based on the way such waves propagate (and in particular the way two component waves reinforce or nullify each other depending on their relative phases) correctly explain measurements of the average way photons travel.

Einstein focused his attention not on the familiar wavelike way radiation propagates, but on what he first realized is the particlelike way it is emitted and absorbed. He reasoned that Planck's requirement that the energy content of the electromagnetic waves of frequency ν in a radiant source (e.g., an ultraviolet light source in a photoelectric experiment) can only be 0, or $h\nu$, or $2h\nu, \ldots,$ or $nh\nu, \ldots$ implies that in the process of going from energy state $nh\nu$ to energy state $(n-1)h\nu$ the source would emit a discrete burst of electromagnetic energy of energy content $h\nu$.

Einstein assumed that such a bundle of energy is initially localized in a small volume of space, and that it remains localized as it moves away from the source with velocity c. He assumed that the energy content E of the bundle, or photon, is related to its frequency ν by the equation

$$E = h\nu \tag{2-2}$$

He also assumed that in the photoelectric process one photon is completely absorbed by one electron in the photocathode.

When the electron is emitted from the surface of the metal, its kinetic energy will be

$$K = h\nu - w \tag{2-3}$$

where $h\nu$ is the energy of the absorbed incident photon and w is the work required to remove the electron from the metal. This work is needed to overcome the attractive fields of the atoms in the surface and losses of kinetic energy due to internal collisions of the electron. Some electrons are bound more tightly than others; some lose energy in collisions on the way out. In the case of loosest binding and no internal losses, the photoelectron will emerge with the maximum kinetic energy, K_{max}. Hence

$$K_{max} = h\nu - w_0 \tag{2-4}$$

where w_0, a characteristic energy of the metal called the *work function,* is the minimum energy needed by an electron to pass through the metal surface and escape the attractive forces that normally bind the electron to the metal.

Consider now how Einstein's photon hypothesis meets the three objections raised against the wave theory interpretation of the photoelectric effect. As for objection 1 (the lack of dependence of K_{max} on the intensity of illumination), there is complete agreement of the photon theory with experiment. Doubling the light intensity merely doubles the number of photons and thus doubles the photoelectric current; it does *not* change the energy $h\nu$ of the individual photons or the nature of the individual photoelectric process described by (2-3).

Objection 2 (the existence of a cutoff frequency) is removed at once by (2-4). If K_{max} equals zero we have

$$h\nu_0 = w_0 \tag{2-5}$$

which asserts that a photon of frequency ν_0 has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If the frequency is reduced below ν_0, the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy individually to eject photoelectrons.

Objection 3 (the absence of a time lag) is eliminated in the photon theory because the required energy is supplied in concentrated bundles. It is *not* spread uniformly over a large area, as we assumed in Example 2-1, which is based on the assumption that the classical wave theory is true. If there is any illumination at all incident on the cathode, then there will be at least one photon that hits it; this photon will be immediately absorbed, by *some* atom, leading to the immediate emission of a photoelectron.

Let us rewrite Einstein's photoelectric equation, (2-4), by substituting eV_0 for K_{max} from (2-1). This yields

$$V_0 = \frac{h\nu}{e} - \frac{w_0}{e}$$

Thus Einstein's theory predicts a linear relationship between the stopping potential V_0 and the frequency ν, in complete agreement with experimental results as shown in Figure 2-3. The slope of the experimental curve in the figure should be h/e or, using data from the figure

$$\frac{h}{e} = \frac{2.1\ \text{V} - 0.1\ \text{V}}{11.0 \times 10^{14}/\text{sec} - 6.0 \times 10^{14}/\text{sec}} = 4.0 \times 10^{-15}\ \text{V-sec}$$

We can find h by multiplying this ratio by the electronic charge e. Thus $h = 4.0 \times 10^{-15}$ V-sec $\times\ 1.6 \times 10^{-19}$ coul $= 6.4 \times 10^{-34}$ joule-sec. From a much more careful analysis of these and other data, including data taken with lithium surfaces, Millikan found the value $h = 6.57 \times 10^{-34}$ joule-sec, with an accuracy of about 0.5%. This early measurement was in good agreement with the value of h derived from Planck's radiation formula. The numerical agreement in two determinations of h, using completely different phenomena and theories, is striking. A modern value of h, deduced from diverse experiments, is

$$h = 6.6262 \times 10^{-34}\ \text{joule-sec}$$

To quote Millikan: "The photoelectric effect . . . furnishes a proof which is quite independent of the facts of blackbody radiation of the correctness of the fundamental assumption of the quantum theory, namely, the assumption of a discontinuous or explosive emission of the energy absorbed by the electronic constituents of atoms from . . . waves. It materializes, so to speak, the quantity h discovered by Planck through the study of blackbody radiation and gives us a confidence inspired by no other type of phenomenon that the primary physical conception underlying Planck's work corresponds to reality."

Example 2-2. Deduce the work function for sodium from Figure 2-3.

►The intersection of the straight line in Figure 2-3 with the horizontal axis is the cutoff frequency, $\nu_0 = 5.6 \times 10^{14}$/sec. Substituting this into (2-5) gives us

$$\begin{aligned} w_0 = h\nu_0 &= 6.63 \times 10^{-34}\ \text{joule-sec} \times 5.6 \times 10^{14}/\text{sec} \\ &= 3.7 \times 10^{-19}\ \text{joule} \times \frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{joule}} \\ &= 2.3\ \text{eV} \end{aligned}$$

The same value is obtained from Figure 2-3 as the magnitude of the intercept of the extended line with the vertical axis.

For most conducting metals the value of the work function is of the order of a few electron volts. It is the same as the work function for thermionic emission from these metals. ◀

Example 2-3. At what rate per unit area do photons strike the metal plate in Example 2-1? Assume that the light is monochromatic, of wavelength 5890 Å (yellow light).

▶The rate per unit area at which energy falls on a metal plate 1 m from a 1-W light source (see Example 2-1) is

$$R = \frac{1 \text{ joule/sec}}{4\pi(1 \text{ m})^2} = 8.0 \times 10^{-2} \text{ joule/m}^2\text{-sec}$$
$$= 5.0 \times 10^{17} \text{ eV/m}^2\text{-sec}$$

Each photon has an energy of

$$E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{ joule-sec} \times 3.00 \times 10^{8} \text{ m/sec}}{5.89 \times 10^{-7} \text{ m}}$$
$$= 3.4 \times 10^{-19} \text{ joule}$$
$$= 2.1 \text{ eV}$$

Thus the rate R at which photons strike a unit area of the plate is

$$R = 5.0 \times 10^{17} \text{ eV/m}^2\text{-sec} \times \frac{1 \text{ photon}}{2.1 \text{ eV}} = 2.4 \times 10^{17} \frac{\text{photon}}{\text{m}^2\text{-sec}}$$

The photoelectric effect is just able to occur because the photon energy just equals the 2.1 eV work function for the potassium surface (see Example 2-1). Note that if the wavelength is slightly increased (that is, if ν is slightly decreased) the photoelectric effect will not occur, no matter how large the rate R might be.

This example suggests that the intensity of light I can be regarded as the product of N, the number of photons per unit area per unit time, and $h\nu$, the energy of a single photon. We see that even at the relatively low intensity here ($\simeq 10^{-1}$ W/m^2) the number N is extremely large ($\simeq 10^{17}$ photons/m^2-sec) so that the energy of any one photon is very small. This accounts for the extreme fineness of the granularity of radiation and suggests why ordinarily it is difficult to detect at all. It is analogous to detecting the atomic structure of bulk matter which for most purposes can be regarded as continuous, the discreteness being revealed only under special circumstances. ◀

In 1921 Einstein received the Nobel Prize for predicting theoretically the law of the photoelectric effect. Before Millikan's complete experimental validation of this law in 1914, Einstein was recommended to membership in the Prussian Academy of Sciences by Planck and others. Their early negative attitude toward the photon hypothesis is revealed in their signed affidavit, praising Einstein, in which they wrote: "Summing up, we may say that there is hardly one among the great problems, in which modern physics is so rich, to which Einstein has not made an important contribution. That he may have sometimes missed the target in his speculations, as, for example, in his hypothesis of light quanta (photons), cannot really be held too much against him, for it is not possible to introduce fundamentally new ideas, even in the most exact sciences, without occasionally taking a risk."

Today the photon hypothesis is used throughout the electromagnetic spectrum, not only in the light region (see Figure 2-4). A microwave cavity, for example, can be said to contain photons. At $\lambda = 10$ cm, a typical microwave wavelength, the photon energy can be computed as above to be 1.20×10^{-5} eV. This energy is much too low to eject photoelectrons from metal surfaces. For x rays, or for energetic γ rays such as are emitted from radioactive nuclei, the photon energy may be 10^6 eV or higher. Such photons can eject electrons bound deep in heavy atoms by energies of the order of 10^5 eV. The photons in the visible region of the electromagnetic spectrum are not energetic enough to do this, the photoelectrons which they eject being the so-called *conduction* electrons which are bound to the metal by energies of only a few electron volts.

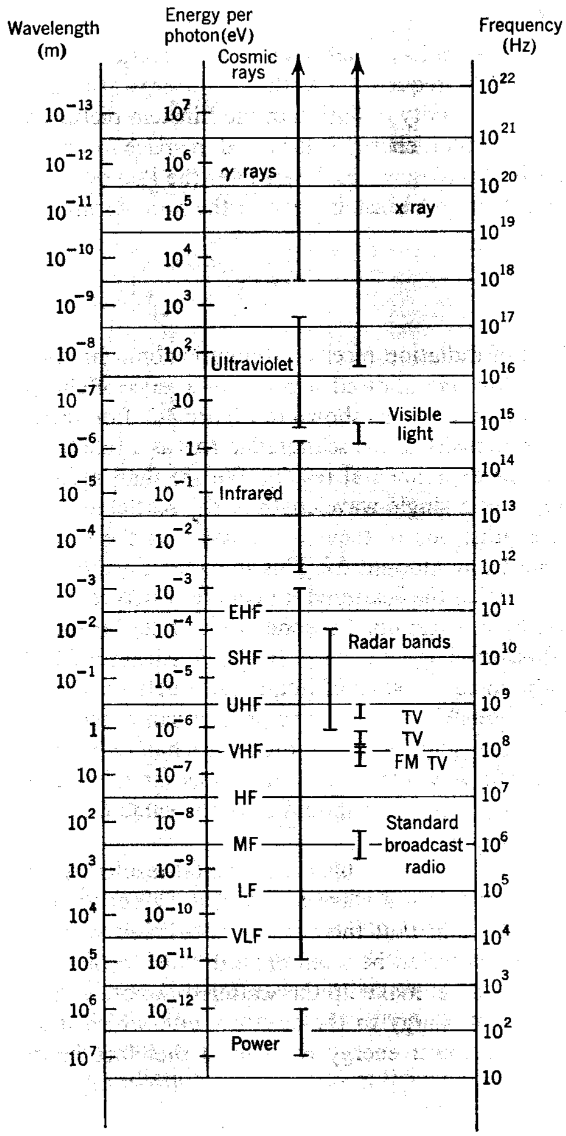

Figure 2-4 The electromagnetic spectrum, showing wavelength, frequency, and energy per photon on a logarithmic scale.

Notice that the photons are absorbed in the photoelectric process. This requires the electrons to be bound to atoms, or solids, for a truly free electron cannot absorb a photon and conserve both total relativistic energy and momentum in the process. We must have a bound electron, therefore, the binding forces serving to transmit momentum to the atom or solid. Due to the large mass of an atom, or solid, compared to the electron, the system can absorb a large amount of momentum without acquiring a significant amount of energy. Our photoelectric energy equation remains valid, the effect being possible only because there is a heavy recoiling particle in addition to an ejected electron. The photoelectric effect is one important way in which photons, of energy up to and including x-ray energies, are absorbed by matter. At higher energies other photon absorption processes, soon to be discussed, become more important.

Finally, it should be emphasized here that in the Einstein picture a photon of frequency ν has exactly the energy $h\nu$; it does *not* have energies that are integral multiples of $h\nu$. Of course, there can be n photons of frequency ν so that the energy at that frequency can be $nh\nu$. In treating blackbody cavity radiation in the Einstein picture, we deal with a "photon gas," because the radiant energy is localized in space in bundles rather than extended through space in standing waves. Years after the Planck deduction of the cavity radiation formula, Bose and Einstein derived the same formula on the basis of a photon gas.

2-4 THE COMPTON EFFECT

The corpuscular (particlelike) nature of radiation received dramatic confirmation in 1923 from the experiments of Compton. He allowed a beam of x rays of sharply defined wavelength λ to fall on a graphite target, as shown in Figure 2-5. For various angles of scattering, he measured the intensity of the scattered x rays as a function of their wavelength. Figure 2-6 shows his experimental results. We see that, although the incident beam consists essentially of a single wavelength λ, the scattered x rays have intensity peaks at *two* wavelengths; one of them is the same as the incident wavelength, the other, λ', being larger by an amount $\Delta\lambda$. This so-called *Compton shift* $\Delta\lambda = \lambda' - \lambda$ varies with the angle at which the scattered x rays are observed.

The presence of scattered wavelength λ' cannot be understood if the incident x radiation is regarded as a classical electromagnetic wave. In the classical model the oscillating electric field vector in the incident wave of frequency ν acts on the free electrons in the scattering target and sets them oscillating at that same frequency. These oscillating electrons, like charges surging back and forth in a small radio transmitting antenna, radiate electromagnetic waves that again have this same frequency ν. Hence, in the classical picture the scattered wave should have the same frequency ν and the same wavelength λ as the incident wave.

Compton (and independently Debye) interpreted his experimental results by postulating that the incoming x-ray beam was not a wave of frequency ν but a collection of photons, each of energy $E = h\nu$, and that these photons collided with free electrons in the scattering target as in a collision between billiard balls. In this view, the "recoil" photons emerging from the target make up the scattered radiation. Since the incident photon transfers some of its energy to the electron with which it collides, the scattered photon must have a lower energy E'; it must therefore have a

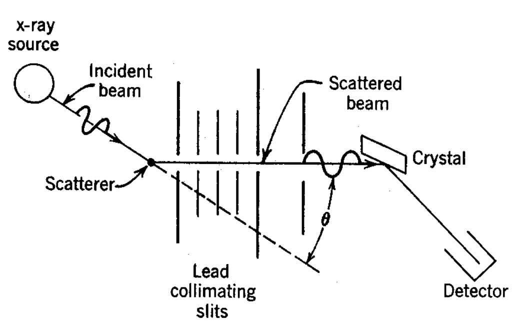

Figure 2-5 Compton's experimental arrangement. Monochromatic x rays of wavelength λ fall on a graphite scatterer. The distribution of intensity with wavelength is measured for x rays scattered at any scattering angle θ. The scattered wavelengths are measured by observing Bragg reflections from a crystal (see Figure 3-3). Their intensities are measured by a detector such as an ionization chamber.